A NONLINEAR CREEP OF PMMA UNDER CYCLIC LOADING

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Abstract

The creep of viscoelastic materials is characterized by a progressive increase in the strain, which occurs even at room temperature, according to both time and applied load. Several works based on experimental tests studied the behavior of creep of these materials under constant stresses. In this work, we study the phenomenon of creep in the case of a periodic loading with constant amplitude. Dynamic tests using PMMA specimen are realized. The results of these tests show that the strain response to an oscillatory excitation of constant amplitude is an oscillatory function, of constant amplitude, around an average strain that evolves in a nonlinear way. In this regard, we developed a theoretical formulation of the nonlinear viscoelastic behavior. This law includes nonlinear factors characterizing the dynamic creep of material. These nonlinear coefficients were determined by dynamic tests on PMMA specimens, under periodic loads with constant amplitudes.

Index Termes: *Polymers, nonlinear viscoelasticity, creep, cyclic loading*

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1. INTRODUCTION

Thermoplastic materials have a nonlinear viscoelastic behavior. Their mechanical properties vary with time and depend on the applied load. In this context, the Boltzmann superposition principle is no more valid to describe the behavior of such viscoelastic nonlinear materials. To do so, several methods were undertaken to determine the laws of viscoelastic nonlinear behavior of these materials, such as the multiple integrals method [1]. However, it has been demonstrated that such method is complex and does not experimentally predict mechanical parameters satisfactorily. In view of this, another method has been developed by Schapery [2-5] based on the principles of the thermodynamics of irreversible processes. This method, which can be viewed as an extension to the non-linear fields of former work of M.A. Biot [6], is exploited and used in non linear viscoelasticity literature.

The constitutive law, initially applied for polymeric materials [7], can indeed be used to study different types of behavior and physical phenomena. Specifically, the constitutive law can be applied for metals at high temperatures [8]. In this context Qingli Dai [9] used Schapery model to establish a coupling damage-viscoelasticity by describing the non linear behavior of asphalt. Martin Lévesque [10] also applied this Schapery model to describe the thermoplastic matrices behavior of composite materials.

In this work, we use the Schapery law to describe the nonlinear dynamic behavior of a viscoelastic material. To achieve this, we use the PMMA to carry out dynamic tests under various periodic loads with constant amplitudes. The results of these tests provide the possibility to determine the nonlinear factors characterizing the dynamic behavior of polymeric materials.

2. DYNAMIC BEHAVIOR LAW

The real behavior of thermoplastic materials is non linear. Based on the thermodynamics irreversible principles, Schapery proposed in his work [2-5] the following non linear law:

$$\varepsilon(\sigma,t) = \sigma\, g_0(\sigma) J_0 + g_1(\sigma)\int_0^t J(t-\tau)\frac{\partial[\sigma\, g_2(\sigma)]}{\partial\tau}d\tau \qquad (1)$$

With $g_0(\sigma)$, $g_1(\sigma)$ and $g_2(\sigma)$ are nonlinear factors, which depend on the stress σ.

$\varepsilon_m^*(t)$

The aim of this work is to study the viscoelastic behavior of a viscoelastic material subjected to a dynamic stress and to determine the nonlinear factors.

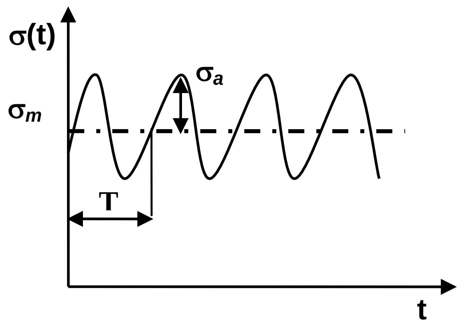

Fig-1: Cyclic excitation

We consider a dynamic stress of pace represented in figure 1, it is decomposed in two parts: σ_m as average of stress and $\sigma_d(t)$ as an excitation loading with a constant amplitude σ_a : This loading can be written in the form:

$$\sigma(t) = \sigma_m + \sigma_d(t) \qquad (2)$$

where: $\sigma_d(t) = \sigma_a f(t)$

and the function f is supposed to be harmonic of period T, which cancels at time $t = 0$. The response to this excitation is represented in figure 2.

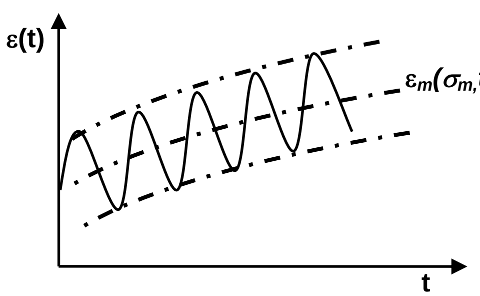

Fig-2: Response to cyclic excitation

In the case of an isothermal transformation, the strain obtained by a dynamic excitation $\sigma_d(t)$ around an average stress σ_m, can be decomposed into an average strain $\varepsilon_m(\sigma_m,t)$ and a dynamic response $\varepsilon_d(\sigma_m,\sigma_a,t)$:

$$\varepsilon(\sigma,t) = \varepsilon_m(\sigma_m,t) + \varepsilon_d(\sigma_m,\sigma_a,t) \qquad (3)$$

$$\varepsilon_m(\sigma_m,t) = \sigma_m g_0(\sigma_m) J_0 + g\ (\sigma_m)\int_0^t J(t-\tau)\frac{\partial[\sigma_m]}{\partial\tau}d\tau \qquad (4)$$

$$\varepsilon_d(\sigma_m,\sigma_a,t) = g\ (\sigma_m,\sigma_a)\int_0^t D(t-\tau)\frac{\partial[\sigma_d(t)]}{\partial\tau}d\tau \qquad (5)$$

Dynamic tests on PMMA specimen (see §3 experimental results) showed that the strain response to an oscillatory excitation of constant amplitude is an oscillatory function of constant amplitude around an average strain. We can express the average strain and the strain due to the dynamic excitation by:

$$\varepsilon_m(\sigma_m,t) = \sigma_m\left[\begin{array}{l} g_0(\sigma_m)J_0 + \\ g_m(\sigma_m)\sum_{i=1}^{m} J_i\left(1-e^{\left(\frac{-t}{\tau_i}\right)}\right)\end{array}\right] \qquad (6)$$

$$\varepsilon_d(\sigma_m,\sigma_a,t) = \sigma_a g_d(\sigma_m,\sigma_a) D\, f(t) \qquad (7)$$

Equation (4) is transformed into equation (6) using the Prony series by introducing the nonlinearity factors $g_0(\sigma_m)$ and $g(\sigma_m)$.

$g_d(\sigma_m,\sigma_a)$ is a nonlinear factor depending on the average stress σ_m and the dynamic excitation amplitude σ_a, which will be given from the results of the dynamic tests. D is a dynamic flexibility representing the linear behavior of material under average stresses σ_m^k and an excitation amplitude relatively low.

Giving ε_a^l the response amplitude to this excitation, we have :

$$D = \frac{\varepsilon_a^l}{\sigma_a^l} \qquad (8)$$

The nonlinear dynamic factor is calculated by the following relationship:

$$g_d(\sigma_m,\sigma_a) = \frac{\varepsilon_a(\sigma_m,\sigma_a)}{\sigma_a D} \qquad (9)$$

The nonlinear factors $g_0(\sigma_m), g_m(\sigma_m)$, and the coefficients of flexibility (J_0, J_i) are determined from the average strain $\varepsilon_m(\sigma_m,t)$ given by dynamic tests on PMMA specimen. To do so, several tests will be carried out under periodic stresses with constant amplitudes, oscillating around the average stress σ_{mk} $(k = 1,..., p)$. For each test (k), we note the deformations $\varepsilon_{mk}^*(t_j)$ associated with times t_j, with $j = 1,..., n$.

The experimental value of the flexibility of creep for the k^{th} test measured at every moment is given by:

$$J_k^*(t_j) = \frac{\varepsilon_{mk}^*(t_j)}{\sigma_{mk}} \qquad (10)$$

For each test k we can write:

$$J_k(t) = g_{0k} J_0 + g_{mk} \sum_{i=1}^{m} J_i \left(1 - e^{\left(\frac{-t}{\tau_i}\right)}\right) \qquad (11)$$

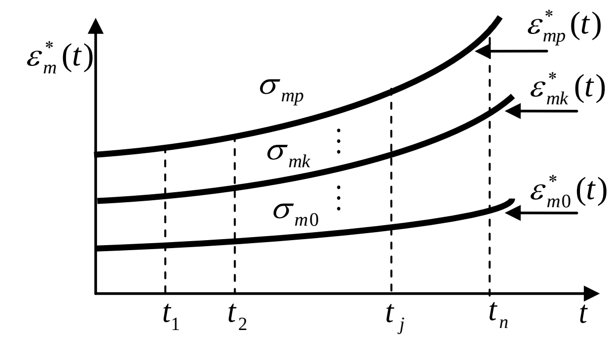

Fig-3: Effect of creep on average strain (*t* in log-scale)

We determine the sum on all times t_j and all the tests k the differences with the squares of the theoretical values $J_k(t)$ and the experimental values $J_k^*(t_j)$ in order to using the least squares methods. This computing technical was used by Brueller [11] to determine the non linearity factors for static creep behavior of polymeric materials.

$$S = \sum_{k=0}^{p} \sum_{j=1}^{n} \left\{ J_k^*(t_j) - g_{0k} J_0 - g_{mk} \sum_{i=1}^{m} J_i \left(1 - e^{\frac{-t}{\tau_i}}\right) \right\}^2 \qquad (12)$$

To minimize the difference between the theoretical and experimental values of rigidities, it is necessary that:

$$\frac{\partial S}{\partial J_i} = 0 \quad \text{for } i = 0, \ldots, m$$

We then obtain a linear system of $(m+1)$ equations which are written in a matrix form as follows:

$$\left[\left(G^t . G\right) \otimes \left(B^t . B\right)\right] \vec{d} = \left[G^t \otimes \left(B^t . J^*\right)\right] \vec{u} \qquad (13)$$

Where the operator $\otimes$ denotes the element- by- element matrices multiplication.

The various matrices are as follows:

$$G = \begin{pmatrix} g_{00} & g_{m0} & \cdots & g_{m0} \\ g_{01} & g_{m1} & \cdots & g_{m1} \\ \cdots & \cdots & \cdots & \cdots \\ g_{0p} & g_{mp} & \cdots & g_{mp} \end{pmatrix}$$, a matrix of dimension $(m+1) \times (p+1)$

$$B = \begin{pmatrix} 1 & \left(1 - e^{\frac{-t_1}{\tau_1}}\right) & \cdots & \left(1 - e^{\frac{-t_1}{\tau_m}}\right) \\ 1 & \left(1 - e^{\frac{-t_2}{\tau_1}}\right) & \cdots & \left(1 - e^{\frac{-t_2}{\tau_m}}\right) \\ \cdots & \cdots & \cdots & \cdots \\ 1 & \left(1 - e^{\frac{-t_n}{\tau_1}}\right) & \cdots & \left(1 - e^{\frac{-t_n}{\tau_m}}\right) \end{pmatrix}$$, a matrix of dimension $(m+1) \times n$

$$J^* = \begin{pmatrix} J_0^*(t_1) & J_1^*(t_1) & \cdots & J_p^*(t_1) \\ J_0^*(t_2) & J_1^*(t_2) & \cdots & J_p^*(t_2) \\ \cdots & \cdots & \cdots & \cdots \\ J_0^*(t_n) & J_1^*(t_n) & \cdots & J_p^*(t_n) \end{pmatrix}$$, a matrix of dimension $(p+1) \times n$

The vectors $\vec{u}$ and $\vec{d}$ are expressed by:

$$\vec{u} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}, \quad \vec{d} = \begin{pmatrix} J_0 \\ J_1 \\ \vdots \\ J_m \end{pmatrix}$$

The method used to solve this system is to suppose that the first test is carried out with low stress and thus one can apply the law of linear behavior. In this case we have :

$$g_{00} = g_{m0} = 1$$

Consequently, the linear flexibility can be written as:

$$J_0^l(t) = J_0 + \sum_{i=1}^{m} J_i \left(1 - e^{\left(\frac{-t}{\tau_i}\right)}\right) \qquad (14)$$

Then we will have:

at time $t = 0$: $J_0^l(0) = J_0$, $\varepsilon_0^l(0) = \sigma_0 J_0$

at time $t=\infty: J_0^l(\infty)=J_0+\sum_{i=1}^{m} J_i$,

$\varepsilon_0^l(\infty)=\sigma_0 J_0^l(\infty)$

For all other tests ($k=1,\dots,p$), the behavior of the material is nonlinear. Flexibilities

At extreme times $t=0$ and $t=t_n\to\infty$ are expressed by:

$$\begin{cases} J_k^n(0)=g_{0k}J_0 \\ J_k^n(\infty)=g_{0k}J_0+g_{mk}\sum_{i=1}^{m}J_i \end{cases} \quad (15)$$

For a first approximation we have

$$\begin{cases} g_{0k}=\dfrac{J_k^0(0)}{J_0^l(0)} \\ g_{mk}=\dfrac{J_k^n(\infty)-J_k^n(0)}{J_0^l(\infty)-J_0^l(0)} \end{cases} \quad (16)$$

In this approach we impose: $t_\infty=t_n$. The strain measured at the moments 0 and t_n are written:

$$\begin{cases} \varepsilon_k^{n*}(t_n)=\sigma_{mk}J_k^n(t_n) \\ \varepsilon_k^{n*}(0)=\sigma_{mk}J_k^n(0) \\ \varepsilon_0^{l*}(t_n)=\sigma_0 J_0^l(t_n) \\ \varepsilon_0^{l*}(0)=\sigma_0 J_0^l(0) \end{cases} \quad (17)$$

We obtain then:

$$\begin{cases} g_{0k}=\dfrac{\varepsilon_k^{n*}(0)\sigma_0}{\varepsilon_0^{l*}(0)\sigma_{mk}} \\ g_{mk}=\dfrac{\left[\varepsilon_k^{n*}(t_n)-\varepsilon_k^{n*}(0)\right]\sigma_0}{\left[\varepsilon_0^{l*}(t_n)-\varepsilon_0^{l*}(0)\right]\sigma_{mk}} \end{cases} \quad (18)$$

Using the method of least squares for each test k, we will have:

$$S_k=\sum_{j=1}^{n}\left\{J_k^*(t_j)-\left[g_{0k}J_0+g_{mk}\sum_{i=1}^{m}J_i\left(1-e^{\left(\frac{-t}{\tau_i}\right)}\right)\right]\right\}^2 \quad (19)$$

The minimization of S_k is obtained when:

$$\begin{cases} \dfrac{\partial S_k}{\partial g_{0k}}=0 \\ \dfrac{\partial S_k}{\partial g_{mk}}=0 \end{cases} \quad (20)$$

The problem is reduced to a matrix form by writing:

$$\left(B_g^t.B_g\right)\vec{g}_k=B_g^t\vec{d}_k^* \quad (21)$$

The vectors $\vec{g}_k$ and $\vec{d}_k$ are as follows:

$$\vec{g}_k=\begin{pmatrix} g_{0k} \\ g_{mk} \end{pmatrix}, \quad \vec{d}_k=\begin{pmatrix} J_k^*(t_1) \\ J_k^*(t_2) \\ \vdots \\ J_k^*(t_n) \end{pmatrix}$$

The matrix B_g of dimension $2\times n$ is written:

$$B_g=\begin{vmatrix} J_0 & \sum_{i=1}^{m}J_i(1-e^{(-t/\tau_i)}) \\ J_0 & \sum_{i=1}^{m}J_i(1-e^{(-t/\tau_i)}) \\ & \vdots \\ J_0 & \sum_{i=1}^{m}J_i(1-e^{(-t/\tau_i)}) \end{vmatrix}$$

With the approximated values g_{0k} and g_{mk} calculated by the relations (18), we determine the flexibilities $J_k(t)$ from equation (11), then using equation (21) we determine new values of g_{0k} and g_{mk}. This iteration is repeated until the convergence of the procedure.

3. EXPERIMENTAL RESULTS

Several dynamic tests were carried out on PMMA specimen test at room temperature (23°C). These tests were carried out on an instrumented machine equipped with force and deformation sensors, (see fig. 4). Various levels of cyclic loading with 2 s period were selected. The average deformations of the dynamic tests and the deformations calculated by the relation (6) are illustrated in figure 5. By the iterative method developed with paragraph 2, and by choosing *m*=5 and *n*=16, the factors of nonlinearity were calculated and illustrated in figure 6. By these factors, a good approximation is reached in describing the nonlinear behavior of test material. A good agreement was noticed between the theoretical and experimental results. In addition the dynamic factor of nonlinearity g_d was calculated from the results provided by dynamic tests using equation (9); Figure 13 clearly shows that g_d depends at the same time on the average stress σ_m and the amplitude of excitation σ_a. It is noted that the amplitude of the deformation, response to an excitation with constant amplitude, remains constant in the course of time. This is due to the small period of the excitation which does not allow the material undergoing creep.

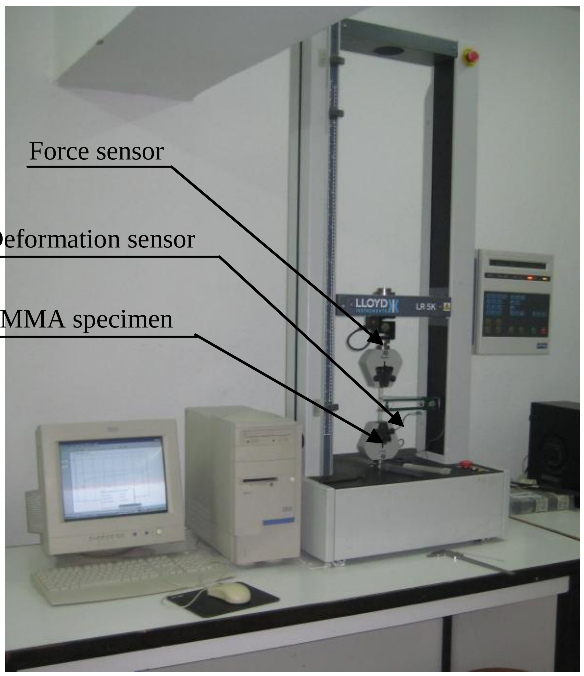

Fig-4: Experimental equipment

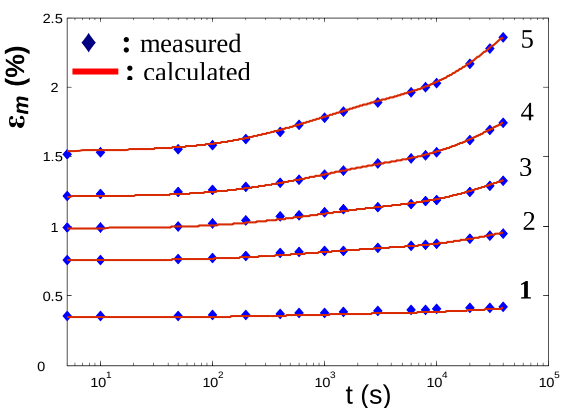

Fig-5: Variation of practical and theoretical average deformation ε_m according to the time in log-scale.
1 : σ_m = 10 MPa , 2 : σ_m = 20 MPa , 3 : σ_m = 25 MPa , 4 : σ_m = 30 MPa , 5 : σ_m = 35 MPa

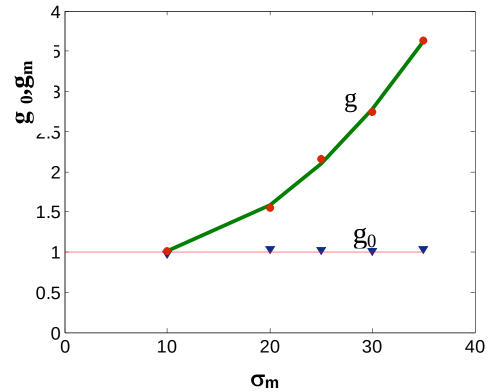

Fig-6: Nonlinear Factors according to the

The dynamic tests with different constant average stresses and various amplitudes were carried out on PMMA specimen test following the standard ISO 527-2 of type 1-A; this has been achieved using a tensile testing machine of the type " LLOYD INSTRUMENT " computer-assisted and controlled by program " NEXYGEN " . The experimental results are illustrated by the figures (7 through 12).

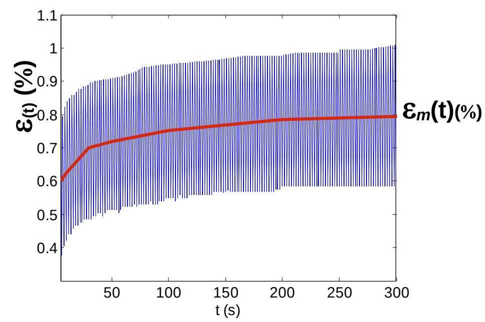

Fig-7: Dynamic deformation for average stress: σ_m= 20MPa , amplitude σ_a = 5 MPa

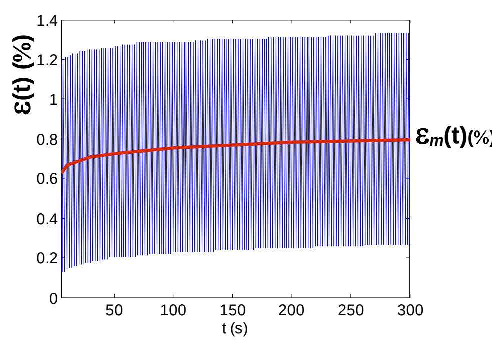

Fig-8: Dynamic deformation for average stress:

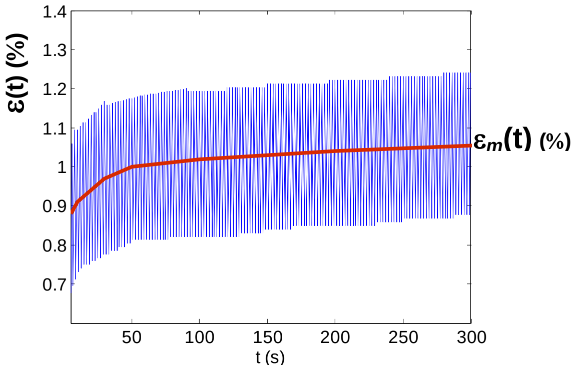

Fig-9: Dynamic deformation for average stress: σ_m= 25 MPa , amplitude σ_a = 5 MPa

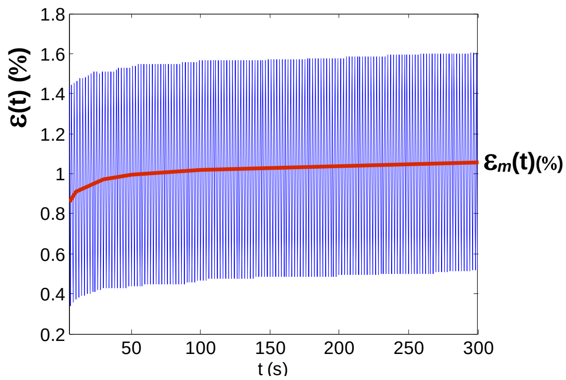

Fig-10: Dynamic deformation for average stress: σ_m= 25 MPa , amplitude σ_a = 15 MPa

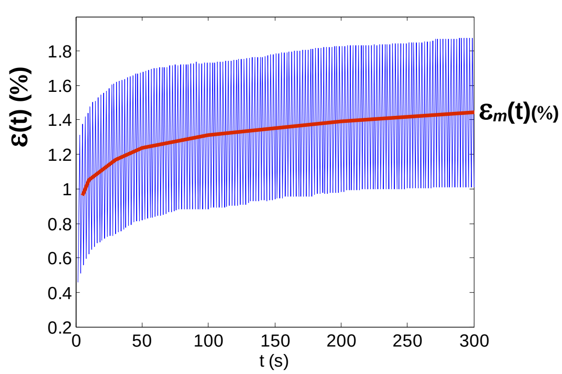

Fig-11: Dynamic deformation for average stress: σ_m= 30 MPa, amplitude σ_a = 10 MPa

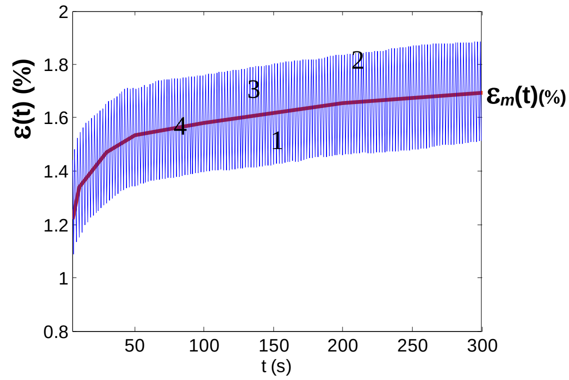

Fig-12: Dynamic deformation for average stress: σ_m= 35 MPa , amplitude σ_a = 5 MPa

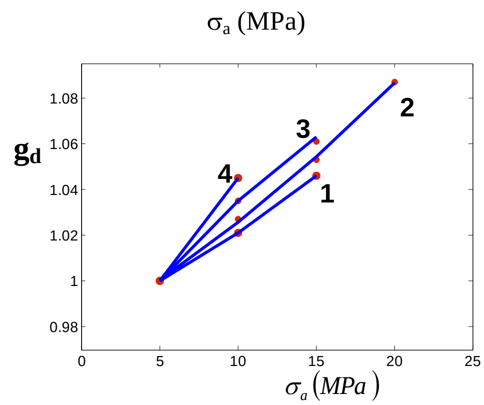

Fig-13: dynamic nonlinear Factors of PMMA specimen test
1 : σ_m= 20 MPa, 2 : σ_m= 25 MPa
3 : σ_m= 30 MPa, 4 : σ_m= 35 MPa

4. CONCLUSION

In this work the dynamic creep behavior of PMMA under periodic loading with constant amplitudes is studied. The dynamic behavior low of this phenomenon is developed using the schapery theory.

The effect of dynamic creep is characterized by nonlinearity factors. They are computed on the basis of a set of dynamic experiments under periodic stress excitations, with constant amplitudes by using least squares techniques.

Using this numerical approach, the PMMA material response under any periodic loading history with constant amplitudes can be predicted.

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